Achievable Rate of Multi-User Mode-Division Multiplexing Using Orbital Angular Momentum

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***Abstract*—This paper investigates achievable rates of a multi-user mode-division multiplexing (MDM) system using *orbital angular momentum* (OAM), where multiple transceivers exploit the same OAM modes. In general, multiple independent signals with different OAM modes are *mutually orthogonal* between perfectly aligned transmitter and receiver antennas, but the OAM signals among multiple users may interfere with each other. Note that we analyze the achievable rate of multi-user MDM system that utilizes the microwave OAM for the first time. Through extensive computer simulations, we analyze the achievable rate of the multi-user OAM-MDM system by considering the multi-user interference. It is worth noting that the communication scenario we consider in this paper has not been investigated in literature so far.**

***Index Terms*—Orbital angular momentum (OAM), mode-division multiplexing (MDM), interference channel, achievable rate, Laguerre-Gauss beam.**

I. Introduction

For supporting explosively increasing wireless data traffic, *orbital angular momentum* (OAM) transmission technique has been considered as one of the emerging technologies especially for line-of-sight (LOS) wireless backhaul networks [1], [2]. In theory, signals with different OAM modes between an coaxially aligned transmitter–receiver pair are known to be *mutually orthogonal*. Most studies on OAM communications consider the OAM-*mode-division multiplexing* (MDM) systems which exploit the orthogonality characteristics among different OAM modes in a single perfectly aligned transmitter–receiver pair [3]–[5]. However, in practice, there may exist multiple OAM transmitter–receiver pairs in dense wireless backhaul environments and the interference among the OAM transmitter–receiver pairs may degrade performance.

In this paper, we investigates the achievable rate of the multi-user OAM mode-division multiplexing (MDM) system, where multiple transmitter–receiver pairs exploit the same OAM modes with Laguerre-Gaussian (LG) beam.

II. System Model

As shown in Fig. 1, there exist K transmitter-receiver pairs and we assume that each pair is coaxially aligned perfectly. Let $\mathcal{L}$ be the set of generated OAM modes at each transmitter, and thus $\mathcal{L} \triangleq \{l_1, l_2, ..., l_m, ..., l_L\}$, where l_m and L denote the m-th OAM mode ($1 \le m \le L$) and the number of OAM modes, respectively. All transmitters are assumed to exploit the same set of OAM modes and each receiver is equipped with L antennas. The OAM channel matrix from the j-th transmitter to the i-th receiver, $\mathbf{H}_{ij} \in \mathbb{C}^{L\times L}$ $(1 \le i, j \le K)$, is given by

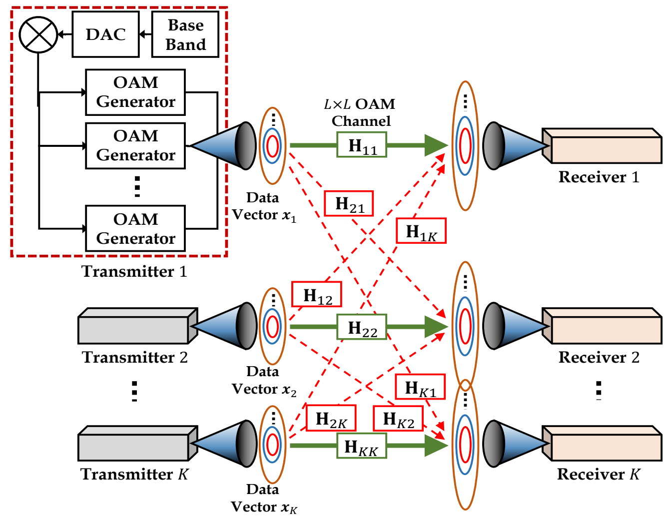

Fig. 1: System model of a multi-user OAM-MDM system

$$\mathbf{H}_{ij} = \begin{bmatrix} h_{ij}^{11} & \cdots & h_{ij}^{1m} & \cdots & h_{ij}^{1L} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{ij}^{n1} & \cdots & h_{ij}^{nm} & \cdots & h_{ij}^{nL} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{ij}^{L1} & \cdots & h_{ij}^{Lm} & \cdots & h_{ij}^{LL} \end{bmatrix}, \tag{1}$$

where the h_{ij}^{nm} $(1 \le n, m \le L)$ represents the OAM channel coefficient from the j-th transmitter with m-th OAM mode to the i-th receiver with the n-th antenna. Then, the received signal at the i-th receiver $\mathbf{y}_i \in \mathbb{C}^{L\times 1}$ is given by

$$\mathbf{y}_i = \sum_{j=1}^{K} \mathbf{H}_{ij}\mathbf{x}_j + \mathbf{z}_i, \tag{2}$$

where the $\mathbf{x}_j \in \mathbb{C}^{L\times 1}$ denotes the transmitted signal with L OAM modes of the j-th transmitter ($\mathbb{E}\left[\|\mathbf{x}_j\|^2\right] = P$) and $\mathbf{z}_i \in \mathbb{C}^{L\times 1}$ denotes the additive white Gaussian noise at the i-th receiver ($\mathbf{z}_i \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$).

In this paper, the OAM signals are assumed to be generated by Laguerre-Gaussian beam with radial mode $p = 0$. The distance between adjacent transmitters and the distance between the transmitter and receiver are assumed to be d_{adj} and

d_{TR}, respectively. In addition, the Rayleigh distance of OAM wave with mode l_m is represented as $z_{R,l_m} = \pi w_{l_m}^2(0)/\lambda$ where λ denotes the wavelength of the OAM wave and $w_{l_m}(0)$ indicates the beam waist of OAM wave with mode l_m at the transmitter ($z = 0$). The beam waist of OAM wave with mode l_m at transmission distance z away from the direction propagating from the transmitter is given by $w_{l_m}(z) = w_{l_m}(0)\sqrt{1+(z/z_{R,l_m})^2}$. The receive antennas for the m-th OAM mode are located at the maximum intensity of the corresponding OAM wave, where the radius of the OAM circle is given by $r_{\mathsf{max}}(z,l_m) = w_{l_m}(z)\sqrt{|l_m|/2}$ [6]. In this paper, we assume that $w_{l_m}(0) = w$ for all m, which implies the same beam waist for all OAM modes at the transmitter. With the cylindrical coordinate system (r,ϕ,z) as in [6], the radial position from the j-th transmitter to the n-th antenna of the i-th receiver at distance d_{TR} is given by

$$r_{ij}^n = \sqrt{((i-j)d_{\mathsf{adj}})^2 + (r_{\mathsf{max}}(d_{\mathsf{TR}}, l_n))^2}, \tag{3}$$

where the receive antennas are assumed to be linearly located on the axis with $\phi = 0$, i.e., y-axis of Cartesian coordinate. Then, the distance between the n-th antenna of the i-th receiver and the j-th transmitter is given by

$$d_{ij}^n = \sqrt{((i-j)d_{\mathsf{adj}})^2 + (r_{\mathsf{max}}(d_{\mathsf{TR}}, l_n))^2 + d_{\mathsf{TR}}^2}. \tag{4}$$

The transverse azimuthal angle between the n-th antenna of the i-th receiver and the j-th transmitter is given by

$$\phi_{ij}^n = \begin{cases} \arctan\left(\frac{r_{\mathsf{max}}(d_{\mathsf{TR}},l_n)}{|i-j|d_{\mathsf{TR}}}\right) & i > j \\ \pi/2 & i = j \\ \pi - \arctan\left(\frac{r_{\mathsf{max}}(d_{\mathsf{TR}},l_n)}{|i-j|d_{\mathsf{TR}}}\right) & i < j. \end{cases} \tag{5}$$

Considering the spatial distribution characteristics of OAM wave, the OAM wireless channel coefficient from the j-th transmitter with OAM wave of mode l_m to the n-th antenna of the i-th receiver, $h_{ij}^{nm} \in \mathbb{C}$, is given by

$$h_{ij}^{nm} = \mathcal{B}\frac{\lambda}{4\pi d_{ii}^n}\left(\frac{r_{ij}^n}{r_{ii}^n}\right)^{|l_m|} e^{-\frac{(r_{ij}^n)^2-(r_{ii}^n)^2}{(w_{l_m}(d_{\mathsf{TR}}))^2}} e^{-ikd_{ii}^n} e^{-i\phi_{ij}^n l_m} \times e^{-i\pi\left(\frac{(r_{ij}^n)^2-(r_{ii}^n)^2}{\lambda d_{\mathsf{TR}}\left(1+\left(\pi(w_{l_m}(d_{\mathsf{TR}}))^2/\lambda d_{\mathsf{TR}}\right)^2\right)}\right)}, \tag{6}$$

where $\mathcal{B}$ denotes the channel gain coefficient [6].

III. Achievable Rate Analysis

With information theory, the achievable sum-rate of the multi-user OAM-MDM system is given by [7]

$$R_{\mathsf{sum}} = \sum_{i=1}^{K}\mathbb{E}\left[\log_2\left(\det\left(\mathbf{I}_L + \frac{P}{N_0}\mathbf{H}_{ii}\mathbf{H}_{ii}^H \times \left(\mathbf{I}_L + \sum_{j\neq i}^{K}\frac{P}{N_0}\mathbf{H}_{ij}\mathbf{H}_{ij}^H\right)^{-1}\right)\right)\right]. \tag{7}$$

Fig. 2 shows the achievable rate of the two-user OAM-MDM system according to the received SNR at the first antenna at each receiver for various system parameters. The carrier frequency is assumed to be 70 GHz and thus $\lambda = 0.004283$ m. We assume that $d_{\mathsf{adj}} = 1$m and $d_{\mathsf{TR}} = 10$m. The achievable rate increases as L or w increases for a given SNR, while it also increases as the SNR increases.

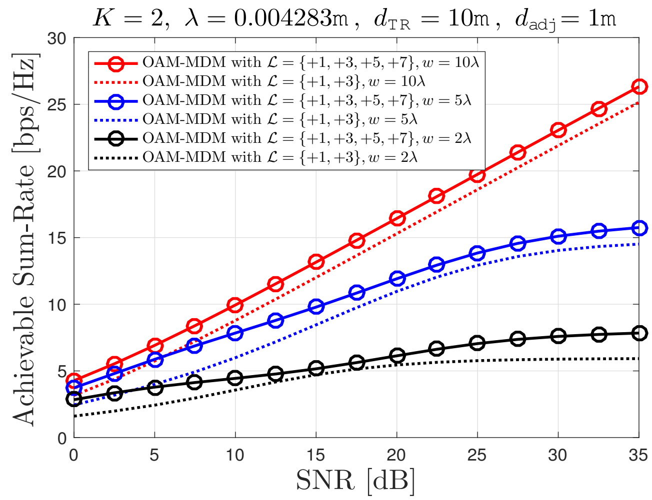

Fig. 2: Achievable rate of the multi-user OAM-MDM system with various system parameters.

IV. Conclusions

We theoretically investigated the achievable rate of the multi-user mode-division multiplexing (MDM) system using the orbital angular momentum (OAM), where each user adopts the same set of OAM modes. The future wireless (fixed) backhaul networks with the OAM transceivers can be modelled by the multi-user OAM-MDM system considered in this paper. We leave the optimal transceiver design for the multi-user OAM-MDM system as a further study.

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